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NUMERICAL SOLUTION OF RELATIVISTIC EQUATIONS FOR BOUND *p*-STATES OF TWO-PARTICLE SYSTEMS

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In this paper numerical solutions are found for quantum field theory integral equations, describing the bound *p*-states of a system of two relativistic spinless particles of mass m each. Considered covariant equations are three-dimensional and have the form [1]

$$G_{0,j}^{-1}(E,E_p)\psi_j(\vec{p}) = \int \frac{-\lambda m \exp(-am\chi_\Delta)}{4\pi\sqrt{(E_pE_k-\vec{p}\vec{k})^2-m^4}}\psi_j(\vec{k})\, m\frac{d\vec{k}}{E_k}, \quad (1)$$

where $\chi_\Delta = \mathrm{Arch}(m^{-2}(E_pE_k-\vec{p}\vec{k}))$ is the rapidity, corresponding to the momentum transfer $\vec{\Delta}$, $\psi_j(\vec{p})$ are the relative motion wave functions, $E_p=\sqrt{\vec{p}^2+m^2}$ and $E_k=\sqrt{\vec{k}^2+m^2}$ are the initial and final energies of the particles respectively, $2E\in[0;2m]$ is the two-particle system's energy, $G_{0,j}^{-1}$ are the inverse Green functions of the Logunov-Tavkhelidze ($j=1$) and Kadyshevsky ($j=2$) equations and of their modified versions ($j=3$, $j=4$):

$$G_{0,1}^{-1}(E,E_p)=E^2-E_p^2;\qquad G_{0,2}^{-1}(E,E_p)=E_p(E-E_p);$$
$$G_{0,3}^{-1}(E,E_p)=m(E^2-E_p^2)/E_p;\quad G_{0,4}^{-1}(E,E_p)=m(E-E_p). \quad (2)$$

Representing the wave functions in the form $\psi_j(\vec{p})=\psi_{j,l}(p)Y_l^\mu(\theta_p,\varphi_p)$, introducing the parameterizations $E=m\cos w$, $p=m\,\mathrm{sh}\chi_p$, $k=m\,\mathrm{sh}\chi_k$ (χ_p, χ_k are the rapidities) and denoting $F_{j,l}(\chi_p)=G_{0,j}^{-1}(E,E_p)\,p\,\psi_{j,l}(p)$, one can obtain partial integral equations

$$F_{j,l}(\chi_p) = -\lambda m^2\int_0^\infty I_l(\chi_p,\chi_k)G_{0,j}(E,m\,\mathrm{ch}\,\chi_k)F_{j,l}(\chi_k)\,d\chi_k. \quad (3)$$

For the case $l=1$ (*p*-states), using the denotations $P(\chi_p)=(1-a^2m^2)^{-1}e^{-am\chi_p}(\mathrm{cth}\chi_p+am)$, $Q(\chi_p)=(am)^{-1}\mathrm{sh}(am\chi_p)\mathrm{cth}\chi_p-\mathrm{ch}(am\chi_p)$, the quantity $I_1(\chi_p,\chi_k)$ can be written in the form

$$I_1(\chi_p,\chi_k)=\begin{cases}P(\chi_k)Q(\chi_p), & \chi_k\ge\chi_p;\\ P(\chi_p)Q(\chi_k), & \chi_k\le\chi_p.\end{cases} \quad (4)$$

Exact analytical solutions of equations (3), can be obtained for $j=1,2$ in the zero mass case of the bound state ($w=\pi/2$), this yields the quantization condition for the coupling constant λ in the form

$$\lambda=(am+2n+2)(am+2n+3). \quad (5)$$

For other values of parameter w solutions were obtained numerically (taking $m=1$). Equation (3) was reduced to a matrix eigenvalue problem after applying the composite trapezium quadrature formula. Accuracy of the solution was enhanced using the Richardson extrapolation [2]. The obtained numerical values of the coupling constant λ have up to six correct significant digits and are in a very good agreement with exact solution (5). Figure 1 illustrates the dependence of the coupling constant λ on parameter $w \in [0;\pi/2]$ with $a = 1, 2, 3$ for the ground state ($n = 0$).

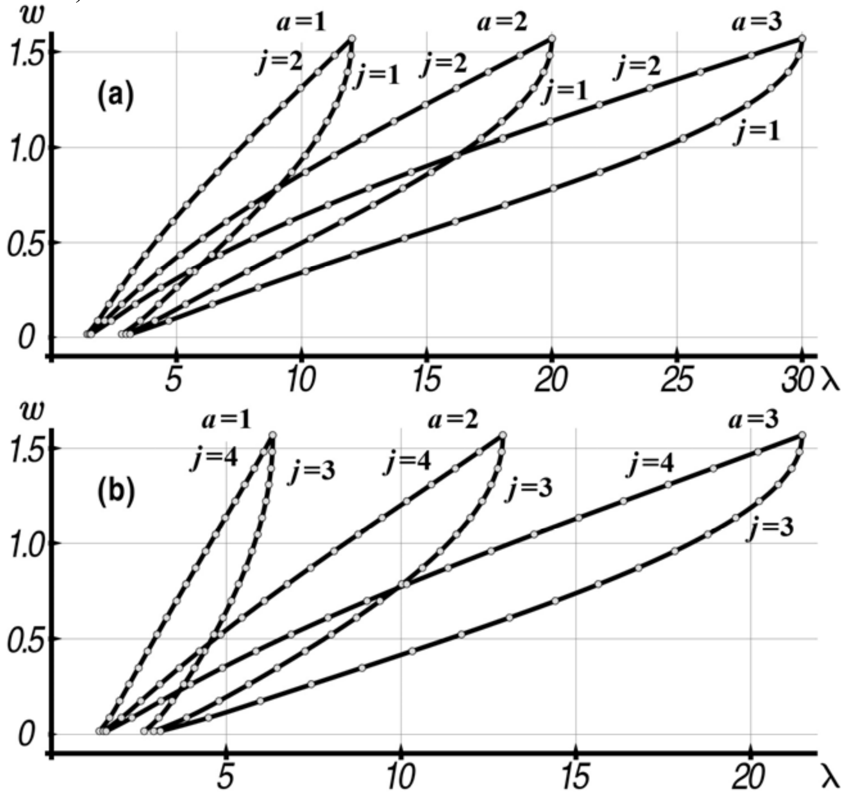

Fig. 1. The coupling constant λ spectrum for $j = 1, 2$ (a) and $j = 3, 4$ (b)

[1] Капшай В. Н. Об одном классе точных решений квазипотенциальных уравнений. / В. Н. Капшай, С. П. Кулешов, Н.Б. Скачков // ТМФ – 1983. – 55. – 3. – С. 349-360.
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