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ЧАСТИЦА СО СПИНОМ 1/2, АНОМАЛЬНЫМ МАГНИТНЫМ МОМЕНТОМ И ПОЛЯРИЗУЕМОСТЬЮ ВО ВНЕШНЕМ МАГНИТНОМ ПОЛЕ

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SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT AND POLARIZABILITY IN THE EXTERNAL MAGNETIC FIELD

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Аннотация. Уравнение для частицы со спином 1/2 и двумя дополнительными характеристиками – аномальным магнитным моментом и поляризуемостью – исследовано при наличии внешнего однородного магнитного поля. После разделения переменных получена система четырех дифференциальных уравнений в полярной координате. Для решения системы применяется метод Федорова – Гронского, основанный на использовании проективных операторов. Согласно этому подходу, четыре полярные компоненты выражаются через две различные функции, последние сводятся к вырожденному гипергеометрическому уравнению; при этом возникает правило квантования. Построены два типа волновых функций, соответствующие энергетические спектры найдены в аналитическом виде и исследованы численно.

Ключевые слова: *частица со спином* 1/2, *дополнительные электромагнитные характеристики*, *магнитное поле*, *проективные операторы*, *точные решения*, *обобщенные энергетические спектры*.

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Abstract. In the present paper, we examine the equation for a spin 1/2 particle with two additional characteristics, anomalous magnetic moment and polarizability in presence of external uniform magnetic field. After separating the variables, we derive the system of four differential equations in the polar coordinate. To resolve the system of equations, we apply the method by Fedorov – Gronskiy, which is based on the use of projective operators. According to this approach, four polar components are expressed through only two different functions, the last reduce to the confluent hypergeometric equation; moreover there arises a definite quantization rule. We have constructed two types of the wave functions, the corresponding energy spectra are found in analytical form. The energy spectra are studied numerically as well.

Keywords: *spin* 1/2 *particle*, *additional electromagnetic characteristics*, *magnetic field*, *projective operators*, *exact solutions*, *generalized energy spectra*.

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1 Introduction

In the paper [1], within the general method by Gel’fand – Yaglom [2], starting with the extended set of representations of the Lorentz group, it was constructed a relativistic generalized equation for a spin 1/2 particle with two additional characteristics (concerning general formalism see in [3]–[5]). After eliminating the accessory variables of the complete wave function, it was derived the generalized Dirac-like equation, the last includes two additional interaction terms which are interpreted as related to anomalous magnetic moment [6]–[9] and a second additional characteristics:

$$\Big\{\gamma^c i(\partial_c + ieA_c) + \frac{e\mu}{2M} j^{[ab]} F_{[ab]} + + \frac{e\sigma}{2M^2}\gamma^c i(\partial_c + ieA_c) j^{[ab]} F_{[ab]} - M\Big\}\Psi = 0; \quad (1.1)$$

the parameter μ corresponds to anomalous magnetic moment of a spin 1/2 particle, and the second parameter σ looks as related to a polarizability of the particle.

In the present paper, we will examine this equation in presence of the external uniform magnetic field.

2 Particle in magnetic field

We will apply the cylindrical coordinates and the tetrad formalism. Let the magnetic field be oriented along the axis z, $A_\phi = eBr^2/2$, $F_{12} = B$. Then the above equation (1.1) takes on the form

$$\left\{ \left[\gamma^0 i\partial_t + \gamma^1\left(\partial_r + \frac{1}{2r}\right) + \frac{\gamma^2}{r}\left(i\partial_\phi - \frac{eBr^2}{2} + ij^{12}\right) + \right.\right.$$
$$\left.\left. + \gamma^3 i\partial_z \right]\left(1 + \frac{e\sigma}{M^2} j^{12} F_{12}\right) + \frac{e\mu}{M} j^{12} F_{12} - M \right\}\Psi = 0.$$

We will apply the Pauli basis for the Dirac matrices

$$\gamma^0 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}, \quad \gamma^1 = \begin{vmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix},$$

$$\gamma^2 = \begin{vmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{vmatrix}, \quad \gamma^3 = \begin{vmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix},$$

$$j^{12} = \frac{1}{2}\gamma^1\gamma^2 = -i\begin{vmatrix} 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{vmatrix},$$

$$ij^{12} = \begin{vmatrix} 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{vmatrix},$$

and the following substitution for the wave function

$$\Psi = \frac{1}{\sqrt{r}} e^{-i\epsilon t} e^{im\phi} e^{ikz} \begin{vmatrix} f_1(r) \\ f_2(r) \\ f_3(r) \\ f_4(r) \end{vmatrix} = \frac{1}{\sqrt{r}} e^{-i\epsilon t} e^{im\phi} e^{ikz} F(r),$$

where $F(r)$ is a 4-dimensional column

$$F(r) = \begin{vmatrix} f_1(r) \\ f_2(r) \\ f_3(r) \\ f_4(r) \end{vmatrix}. \qquad (2.1)$$

Let us (temporally) simplify the notations

$$eB \Rightarrow B, \quad eF_{12} \Rightarrow B, \quad e\mu \Rightarrow \mu, \quad e\sigma \Rightarrow \sigma;$$

correspondingly, the main equation reads

$$\left\{ \left[\epsilon\gamma^0 + i\gamma^1\partial_r + \frac{\gamma^2}{r}\left(-m - \frac{Br^2}{2} + ij^{12}\right) - k\gamma^3 \right] \times \right.$$
$$\left. \times \left(1 + \frac{\sigma B}{M^2} j^{12}\right) + \frac{\mu B}{M} j^{12} - M \right\}\Psi = 0,$$

or

$$\left\{ \left[\begin{vmatrix} 0 & 0 & \epsilon & 0 \\ 0 & 0 & 0 & \epsilon \\ \epsilon & 0 & 0 & 0 \\ 0 & \epsilon & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 0 & -i\partial_r \\ 0 & 0 & -i\partial_r & 0 \\ 0 & i\partial_r & 0 & 0 \\ i\partial_r & 0 & 0 & 0 \end{vmatrix} + \right.\right.$$

$$+ \begin{vmatrix} 0 & 0 & 0 & \frac{i}{r} \\ 0 & 0 & -\frac{i}{r} & 0 \\ 0 & -\frac{i}{r} & 0 & 0 \\ \frac{i}{r} & 0 & 0 & 0 \end{vmatrix} \times$$

$$\times \begin{vmatrix} -m + \frac{1}{2} - \frac{Br^2}{2} & 0 & 0 & 0 \\ 0 & -m - \frac{1}{2} - \frac{Br^2}{2} & 0 & 0 \\ 0 & 0 & -m + \frac{1}{2} - \frac{Br^2}{2} & 0 \\ 0 & 0 & 0 & -m - \frac{1}{2} - \frac{Br^2}{2} \end{vmatrix} -$$

$$\left. -k\begin{vmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix} \right] \times$$

$$\times \left(1 - i\frac{\sigma B}{M^2}\begin{vmatrix} 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{vmatrix}\right) -$$

$$\left. -i\frac{\mu B}{M}\begin{vmatrix} 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{vmatrix} - M \right\}\Psi = 0.$$

With the use of notations

$$a_{m+1/2} = \frac{d}{dr} + \frac{m + 1/2 + Br^2/2}{r},$$

$$b_{m-1/2} = \frac{d}{dr} - \frac{m - 1/2 + Br^2/2}{r},$$

the last equation reads simpler

$$\begin{vmatrix} 0 & 0 & (\epsilon + k) & 0 \\ 0 & 0 & 0 & (\epsilon - k) \\ (\epsilon - k) & 0 & 0 & 0 \\ 0 & (\epsilon + k) & 0 & 0 \end{vmatrix} \begin{vmatrix} \left(1 - i\frac{\sigma B}{2M^2}\right) f_1(r) \\ \left(1 + i\frac{\sigma B}{2M^2}\right) f_2(r) \\ \left(1 - i\frac{\sigma B}{2M^2}\right) f_3(r) \\ \left(1 + i\frac{\sigma B}{2M^2}\right) f_4(r) \end{vmatrix} +$$

$$+\begin{vmatrix} 0 & 0 & 0 & -ia_{m+1/2} \\ 0 & 0 & -ib_{m-1/2} & 0 \\ 0 & ia_{m+1/2} & 0 & 0 \\ ib_{m-1/2} & 0 & 0 & 0 \end{vmatrix} \times$$

$$\times \begin{vmatrix} \left(1+i\frac{\sigma B}{2M^2}\right) f_1(r) \\ \left(1-i\frac{\sigma B}{2M^2}\right) f_2(r) \\ \left(1+i\frac{\sigma B}{2M^2}\right) f_3(r) \\ \left(1-i\frac{\sigma B}{2M^2}\right) f_4(r) \end{vmatrix} + \begin{vmatrix} \left(-i\frac{\mu B}{2M}-M\right) f(r)_1 \\ \left(+i\frac{\mu B}{2M}-M\right) f_2(r) \\ \left(-i\frac{\mu B}{2M}-M\right) f_3(r) \\ \left(+i\frac{\mu B}{2M}-M\right) f_4(r) \end{vmatrix} = 0,$$

it leads to the separate equations

$$-a_{m+1/2} f_4(r)\left(\frac{B\sigma}{2M^2}+i\right)+ +f_3(r)(k+\varepsilon)\left(1-\frac{iB\sigma}{2M^2}\right)+f_1(r)\left(-\frac{iB\mu}{2M}-M\right)=0,$$

$$b_{m-1/2} f_3(r)\left(\frac{B\sigma}{2M^2}-i\right)+ +f_4(r)(\varepsilon-k)\left(1+\frac{iB\sigma}{2M^2}\right)+f_2(r)\left(+\frac{iB\mu}{2M}-M\right)=0,$$

$$a_{m+1/2} f_2(r)\left(\frac{B\sigma}{2M^2}+i\right)+ +f_1(r)(\varepsilon-k)\left(1-\frac{iB\sigma}{2M^2}\right)+f_3(r)\left(-\frac{iB\mu}{2M}-M\right)=0,$$

$$-b_{m-1/2} f_1(r)\left(\frac{B\sigma}{2M^2}-i\right)+ +f_2(r)(\varepsilon+k)\left(1+\frac{iB\sigma}{2M^2}\right)+f_4(r)\left(+\frac{iB\mu}{2M}-M\right)=0. \quad (2.2)$$

In order to resolve this system, we will apply the method by Fedorov – Gronskiy [10]. It is based on the use of projective operators related to the third spin projection

$$Y=ij^{12}=\begin{vmatrix} 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{vmatrix}; P_+=\begin{vmatrix} 1000 \\ 0000 \\ 0010 \\ 0000 \end{vmatrix}, P_-=\begin{vmatrix} 0000 \\ 0100 \\ 0000 \\ 0001 \end{vmatrix};$$

according to this approach, each projective constituent is determined through one function:

$$\Psi_+(r)=\begin{vmatrix} f_1 \\ 0 \\ f_3 \\ 0 \end{vmatrix} F_1(r), \quad \Psi_-(r)=\begin{vmatrix} 0 \\ f_2 \\ 0 \\ f_4 \end{vmatrix} F_2(r),$$

where $F_1(r), F_2(r)$ note two main functions from which functions $f_1(r), f_2(r), f_3(r), f_4(r)$ are constructed (see (2.1)); f_1, f_2, f_3, f_4 stand for numerical variables that will be found below.

Therefore, equations (2.2) take on the form

$$-a_{m+1/2} f_4 F_2(r)\left(\frac{B\sigma}{2M^2}+i\right)+ +f_3 F_1(r)(k+\varepsilon)\left(1-\frac{iB\sigma}{2M^2}\right)+f_1 F_1(r)\left(-\frac{iB\mu}{2M}-M\right)=0,$$

$$b_{m-1/2} f_3 F_1(r)\left(\frac{B\sigma}{2M^2}-i\right)+ +f_4 F_2(r)(\varepsilon-k)\left(1+\frac{iB\sigma}{2M^2}\right)+f_2 F_2(r)\left(+\frac{iB\mu}{2M}-M\right)=0,$$

$$a_{m+1/2} f_2 F_2(r)\left(\frac{B\sigma}{2M^2}+i\right)+ +f_1 F_1(r)(\varepsilon-k)\left(1-\frac{iB\sigma}{2M^2}\right)+f_3 F_1(r)\left(-\frac{iB\mu}{2M}-M\right)=0,$$

$$-b_{m-1/2} f_1 F_1(r)\left(\frac{B\sigma}{2M^2}-i\right)+ +f_2 F_2(r)(\varepsilon+k)\left(1+\frac{iB\sigma}{2M^2}\right)+f_4 F_2(r)\left(+\frac{iB\mu}{2M}-M\right)=0.$$

Also, we should impose differential constraints that will permit us to transform differential equations into algebraic ones:

$$-a_{m+1/2} f_4 F_2(r)\left(\frac{B\sigma}{2M^2}+i\right)+ +f_3 F_1(r) f(k+\varepsilon)\left(1-\frac{iB\sigma}{2M^2}\right)+ +f_1 F_1(r)\left(-\frac{iB\mu}{2M}-M\right)=0, \quad a_{m+1/2}F_2(r)=C_1F_1;$$

$$b_{m-1/2} f_3 F_1(r)\left(\frac{B\sigma}{2M^2}-i\right)+f_4 F_2(r)(\varepsilon-k)\left(1+\frac{iB\sigma}{2M^2}\right)+ +f_2 F_2(r)\left(+\frac{iB\mu}{2M}-M\right)=0, \quad b_{m-1/2}F_1(r)=C_2F_2;$$

$$a_{m+1/2} f_2 F_2(r)\left(\frac{B\sigma}{2M^2}+i\right)+f_1 F_1(r)(\varepsilon-k)\left(1-\frac{iB\sigma}{2M^2}\right)+ +f_3 F_1(r)\left(-\frac{iB\mu}{2M}-M\right)=0, \quad a_{m+1/2}F_2(r)=C_1F_1;$$

$$-b_{m-1/2} f_1 F_1(r)\left(\frac{B\sigma}{2M^2}-i\right)+f_2 F_2(r)(\varepsilon+k)\left(1+\frac{iB\sigma}{2M^2}\right)+ +f_4 F_2(r)\left(+\frac{iB\mu}{2M}-M\right)=0, \quad b_{m-1/2}F_1(r)=C_2F_2.$$

Taking into account these two constraints

$$a_{m+1/2}F_2(r)=C_1F_1, \quad b_{m-1/2}F_1(r)=C_2F_2,$$

we get the algebraic system

$$-C_1\left(\frac{B\sigma}{2M^2}+i\right)f_4+(k+\varepsilon)\left(1-\frac{iB\sigma}{2M^2}\right)f_3+ +\left(-\frac{iB\mu}{2M}-M\right)f_1=0,$$

$$C_2\left(\frac{B\sigma}{2M^2}-i\right)f_3+(\varepsilon-k)\left(1+\frac{iB\sigma}{2M^2}\right)f_4+ +\left(+\frac{iB\mu}{2M}-M\right)f_2=0,$$

$$C_1\left(\frac{B\sigma}{2M^2}+i\right)f_2+(\varepsilon-k)\left(1-\frac{iB\sigma}{2M^2}\right)f_1+$$

$$+\left(-\frac{iB\mu}{2M}-M\right)f_3=0,$$

$$-C_2\left(\frac{B\sigma}{2M^2}-i\right)f_1+(\varepsilon+k)\left(1+\frac{iB\sigma}{2M^2}\right)f_2+$$

$$+\left(+\frac{iB\mu}{2M}-M\right)f_4=0.$$

Without loss of generality, we can equate two parameters, $C_2=C_1=C$, so we obtain

$$a_{m+1/2}F_2(r)=CF_1,\ b_{m-1/2}F_1(r)=CF_2\Rightarrow$$

$$(a_{m+1/2}b_{m-1/2}-C^2)F_1(r)=0,$$

$$(b_{m-1/2}a_{m+1/2}-C^2)F_2(r)=0;$$

then the above algebraic system reads simpler

$$-\left(\frac{iB\mu}{2M}+M\right)f_1+0\cdot f_2+(k+\varepsilon)\left(1-\frac{iB\sigma}{2M^2}\right)f_3-$$

$$-C\left(\frac{B\sigma}{2M^2}+i\right)f_4=0,$$

$$0\cdot f_1+\left(\frac{iB\mu}{2M}-M\right)f_2+C\left(\frac{B\sigma}{2M^2}-i\right)f_3+$$

$$+(\varepsilon-k)\left(1+\frac{iB\sigma}{2M^2}\right)f_4=0,$$

$$(\varepsilon-k)\left(1-\frac{iB\sigma}{2M^2}\right)f_1+C\left(\frac{B\sigma}{2M^2}+i\right)f_2-$$

$$-\left(\frac{iB\mu}{2M}+M\right)f_3+0\cdot f_4=0,$$

$$-C\left(\frac{B\sigma}{2M^2}-i\right)f_1+(\varepsilon+k)\left(1+\frac{iB\sigma}{2M^2}\right)f_2+$$

$$+0\cdot f_3+\left(\frac{iB\mu}{2M}-M\right)f_4=0. \qquad (2.3)$$

3 Solving the second order equations for $F_1(r)$ and $F_2(r)$

We have two equations

$$(a_{m+1/2}b_{m-1/2}-C^2)F_1(r)=0,$$

$$(b_{m-1/2}a_{m+1/2}-C^2)F_2(r)=0,$$

where

$$a_{m+1/2}=\frac{d}{dr}+\frac{m+1/2+Br^2/2}{r},$$

$$b_{m-1/2}=\frac{d}{dr}-\frac{m-1/2+Br^2/2}{r}.$$

In explicit form, these equations read

$$\frac{d^2F_1}{dr^2}+\frac{1}{r}\frac{dF_1}{dr}+$$

$$+\left[-\frac{1}{4}B^2r^2-\frac{1}{2}B-mB-C^2-\frac{(m-1/2)^2}{r^2}\right]F_1=0,$$

$$\frac{d^2F_2}{dr^2}+\frac{1}{r}\frac{dF_2}{dr}+$$

$$+\left[-\frac{1}{4}B^2r^2+\frac{1}{2}B-mB-C^2-\frac{(m+1/2)^2}{r^2}\right]F_2=0.$$

Let us transform them to another variable, $x=-Br^2/2$:

$$\frac{d^2F_1}{dx^2}+\frac{1}{x}\frac{dF_1}{dx}+$$

$$+\left[-\frac{1}{4}+\frac{1}{4}\frac{B+2mB+2C^2}{Bx}-\frac{1}{4}\frac{(1/2-m)^2}{x^2}\right]F_1=0,$$

$$\frac{d^2F_2}{dx^2}+\frac{1}{x}\frac{dF_2}{dx}+$$

$$+\left[-\frac{1}{4}+\frac{1}{4}\frac{-B+2mB+2C^2}{Bx}-\frac{1}{4}\frac{(m+1/2)^2}{x^2}\right]F_2=0.$$

These two equations are related by the simple symmetry

$$B\Rightarrow -B,\ m\Rightarrow -m,\ F_1\Rightarrow F_2;$$

so it suffices to solve the equation for $F_1(x)$:

$$F_1(x)=x^Ae^{Dx}f(x),$$

$$f''+\left(\frac{2A+1}{x}+2D\right)f'+\frac{A^2}{x^2}f-\frac{1}{4}\frac{(1/2-m)^2}{x^2}f+$$

$$+D^2f-\frac{1}{4}f+\frac{(2A+1)D}{x}f+\frac{1}{4}\frac{1+2m+2C^2/B}{x}f=0.$$

Let us impose the constraints

$$A^2=\frac{1}{4}(1/2-m)^2,\ D^2=\frac{1}{4},$$

in order to have solutions finite at two singular points, we should use

$$A=\frac{|m-1/2|}{2},\ D=1/2\ \text{(let } B>0).$$

In this way, we arrive at a simpler equation

$$x\frac{d^2f}{dx^2}+(2A+1+x)\frac{df}{dx}+$$

$$+\left[\left(A+\frac{1}{2}\right)+\frac{1}{4}\left(1+2m+\frac{2C^2}{B}\right)\right]f=0.$$

In the variable $y=-x$, we get a confluent hypergeometric equation

$$y\frac{d^2f}{dt^2}+(2A+1-y)\frac{df}{dy}-$$

$$-\left[\left(A+\frac{1}{2}\right)+\frac{1}{4}\left(1+2m+\frac{2C^2}{B}\right)\right]f=0,$$

with parameters

$$c=2A+1=|m-1/2|+1,$$

$$a=\left(A+\frac{1}{2}\right)+\frac{1}{4}\left(1+2m+\frac{2C^2}{B}\right)=$$

$$=\frac{|m-1/2|+m+1/2}{2}+\frac{C^2}{2B}+\frac{1}{2}.$$

The polynomial condition $a=-n$ gives the following quantization rule

$$C^2 = -2B\left(n + \frac{|m-1/2| + m + 1/2}{2} + \frac{1}{2}\right), \quad (3.1)$$

$$n = 0, 1, 2, ...$$

Relevant solutions for the second function $F_2(z)$ may be found by using the first order relation

$$b_{m-1/2} F_1(r) = C F_2,$$

$$b_{m-1/2} = \frac{d}{dr} - \frac{m - 1/2 + Br^2/2}{r}.$$

4 Helicity operator

Let us turn to the differential system (2.2) and try to impose the linear constraints

$$f_3 = Af_1, \; f_4 = Af_2;$$

in the case of ordinary Dirac particle, they relate to diagonalization of the helicity operator. Then, collecting equations in two pairs we have

$$-a_{m+1/2} A f_2 \left(\frac{B\sigma}{2M^2} + i\right) +$$
$$+ f_1 \left[A(k+\varepsilon)\left(1 - \frac{iB\sigma}{2M^2}\right) - \left(\frac{iB\mu}{2M} + M\right)\right] = 0,$$

$$a_{m+1/2} f_2 \left(\frac{B\sigma}{2M^2} + i\right) +$$
$$+ f_1 \left[(\varepsilon - k)\left(1 - \frac{iB\sigma}{2M^2}\right) - A\left(\frac{iB\mu}{2M} + M\right)\right] = 0,$$

$$b_{m-1/2} A f_1 \left(\frac{B\sigma}{2M^2} - i\right) +$$
$$+ f_2 \left[A(\varepsilon - k)\left(1 + \frac{iB\sigma}{2M^2}\right) + f_2\left(\frac{iB\mu}{2M} - M\right)\right] = 0,$$

$$-b_{m-1/2} f_1 \left(\frac{B\sigma}{2M^2} - i\right) +$$
$$+ f_2 \left[(\varepsilon + k)\left(1 + \frac{iB\sigma}{2M^2}\right) + A\left(\frac{iB\mu}{2M} - M\right)\right] = 0.$$

For consistency, we should expect that both equations are valid:

$$A(k+\varepsilon)\left(1 - \frac{iB\sigma}{2M^2}\right) - \left(\frac{iB\mu}{2M} + M\right) =$$
$$= -A\left[(\varepsilon - k)\left(1 - \frac{iB\sigma}{2M^2}\right) - A\left(\frac{iB\mu}{2M} + M\right)\right],$$

$$A(\varepsilon - k)\left(1 + \frac{iB\sigma}{2M^2}\right) + \left(\frac{iB\mu}{2M} - M\right) =$$
$$= -A\left[(\varepsilon + k)\left(1 + \frac{iB\sigma}{2M^2}\right) + A\left(\frac{iB\mu}{2M} - M\right)\right].$$

However, we immediately see that these equations are different; this means that the above linear conditions do not agree with the system of four equations. Therefore, the helicity operator cannot be diagonalized for this generalized Dirac-like equation.

5 Solving the algebraic system, the energy spectra

Let us turn again to the above algebraic system (2.3). It is convenient to introduce dimensionless quantities:

$$\frac{\varepsilon}{M} = E, \; \frac{k}{M} = K, \; \frac{C}{M} = c, \; b = \frac{B}{2M^2},$$

$$\frac{iB\sigma}{2M^2} = ib\sigma, \; \frac{iB\mu}{2M^2} = ib\mu;$$

then the system transforms to

$$-(ib\mu + 1) f_1 + 0 \cdot f_2 +$$
$$+ (E+K)(1 - ib\sigma) f_3 - c(b\sigma + i) f_4 = 0,$$
$$0 \cdot f_1 + (ib\mu - 1) f_2 + c(b\sigma - i) f_3 +$$
$$+ (E-K)(1 + ib\sigma) f_4 = 0,$$
$$(E-K)(1 - ib\sigma) f_1 + c(b\sigma + i) f_2 -$$
$$-(ib\mu + 1) f_3 + 0 \cdot f_4 = 0,$$
$$-c(bB\sigma - i) f_1 + (E+K)(1 + ib\sigma) f_2 +$$
$$+ 0 \cdot f_3 + (ib\mu - 1) f_4 = 0,$$

or in matrix form

$$\begin{vmatrix} -(ib\mu+1) & 0 & (E+K)\times(1-ib\sigma) & -c(b\sigma+i) \\ 0 & ib\mu-1 & c(b\sigma-i) & (E-K)\times(1+ib\sigma) \\ (E-K)\times(1-ib\sigma) & c(b\sigma+i) & -(ib\mu+1) & 0 \\ -c(b\sigma-i) & (E+K)\times(1+ib\sigma) & 0 & ib\mu-1 \end{vmatrix} \begin{vmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{vmatrix} = 0. \quad (5.1)$$

From vanishing its determinant, we derive a bi-quadratic equation

$$\det A = b^4\left[\left(E^2 - K^2 + c^2\right)\sigma^2 - \mu^2\right]^2 +$$
$$+ \left(E^2 - K^2 + c^2 - 1\right)^2 + b^2\left\{2\left(E^2 - K^2 - c^2 + 1\right)\mu^2 +\right.$$
$$+ 2\left[\left(E^2 - K^2 + c^2\right)^2 + E^2 - K^2 - c^2\right]\sigma^2 +$$
$$\left. + 8\left(E^2 - K^2\right)\sigma\mu\right\} = 0.$$

To separate the contribution of the additional characteristics μ and σ, let us introduce a new variable $x = E^2 - K^2 - 1 + c^2$; when $\mu = 0, \; \sigma = 0$, the energy values are given by equation

$$x = E^2 - K^2 - 1 + c^2 = 0 \;\Rightarrow\; E^2 = 1 + K^2 - c^2 > 0.$$

In general case $(\mu \neq 0, \; \sigma \neq 0)$, we get the quadratic equation with respect to x:

$$x^2 - 2\frac{b^2(\sigma+\mu)\left(-b^2\sigma^3 + b^2\mu\sigma^2 - 3\sigma - \mu\right)}{\left(1 + b^2\sigma^2\right)^2} x +$$
$$+ \frac{b^2(\sigma+\mu)^2\left(b^2\mu^2 - 2b^2\mu\sigma + b^2\sigma^2 - 4c^2 + 4\right)}{\left(1 + b^2\sigma^2\right)^2} = 0.$$

We readily find its roots

$$x = E^2 - K^2 - 1 + c^2 =$$
$$= \frac{b(\sigma+\mu)}{(1+b^2\sigma^2)^2}\Big[\big(\sigma^2(\mu-\sigma)b^2 - \mu - 3\sigma\big)b \pm$$
$$\pm 2\sqrt{\big(\sigma(c\sigma+\mu)b^2 + c - 1\big)\big(\sigma(c\sigma-\mu)b^2 + c + 1\big)}\Big] > 0.$$

Taking into account for E:

$$E_{1,2} = E_{\pm} =$$
$$= \frac{1}{1+b^2\sigma^2}\Big[\pm 2b(\sigma+\mu)\sqrt{c^2(1+b^2\sigma^2)^2 - (b^2\mu\sigma - 1)^2} +$$
$$+\big((-c^2+K^2)\sigma^4 + \sigma^2\mu^2\big)b^4 +$$
$$+\big((-1-2c^2+2K^2)\sigma^2 - 4\sigma\mu - \mu^2\big)b^2 - c^2 + 1 + K^2\Big]^{1/2}$$

in the system (5.1), we can find two types of the wave functions.

6 Numerical study of the spectra

The energy spectra depend in a complicated way on additional characteristics; for this reason these spectra should be studied numerically. Let us write down the values of energies for three sets of parameters.

In numerical calculations we must make the changes $\sigma \to i\sigma$, $\mu \to i\mu$, they lead to physically interpretable results. The new parameters σ, μ are real. In the paper, complex factors were not added at the beginning for the reasons of brevity.

Let us calculate the energy values for fixed values of parameter N, which is determined by the relation

$$N = n + \frac{|m-1/2| + m + 1/2}{2} + \frac{1}{2},$$

where m is the value of the third projection of the total angular momentum. The parameter N enters into expression (3.1) for the parameter C, which is quantized due to the presence of the external magnetic field.

Table 6.1 − Calculation of energies E_+, E_- for parameters $\sigma = 0,02i$, $\mu = 0,02i$, $K = 0,01$, $b = 0,2$, $\sigma, \mu << b$

N	E_+	E_-
0	0,992082273	1,008081729
1	1,175285418	1,191285103
2	1,333702000	1,349701812
3	1,475295037	1,491294929
4	1,604502430	1,620502379
5	1,724098156	1,740098146
6	1,835953352	1,851953373
7	1,941400911	1,957400957
8	2,041430133	2,057430199
9	2,136799251	2,152799333
10	2,228104602	2,244104698

Table 6.2 − Calculation of energies E_+, E_- for parameters $\sigma = 0$, $\mu = 0,02i$, $K = 0,01$, $b = 0,2$, $\sigma, \mu << b$

N	E_+	E_-
0	0,996050200	1,004049800
1	1,179258357	1,187258071
2	1,337678165	1,345677943
3	1,479273498	1,487273317
4	1,608482635	1,616482481
5	1,728079742	1,736079608
6	1,839936067	1,847935949
7	1,945384571	1,953384466
8	2,045414598	2,053414503
9	2,140784415	2,148784328
10	2,232090378	2,240090298

For comparing, it has a sense to get similar table 6.3 for the energies at vanishing additional parameters:

Table 6.3 − Calculation of energies E_+, E_- for parameters $\sigma = 0$, $\mu = 0$, $K = 0,01$, $b = 0,2$, $\sigma, \mu << b$

N	E_+	E_-
0	1,000049999	1,000049999
1	1,183258214	1,183258214
2	1,341678054	1,341678054
3	1,483273407	1,483273407
4	1,612482558	1,612482558
5	1,732079675	1,732079675
6	1,843936008	1,843936008
7	1,949384518	1,949384518
8	2,049414551	2,049414551
9	2,144784371	2,144784371
10	2,236090338	2,236090338

Conclusion

In the present paper, we have examined the Dirac-like equation for a spin 1/2 particle with two additional characteristics, anomalous magnetic moment μ and polarizability σ in presence of external uniform magnetic field. After separating the variables, we derived the system of four differential equations in the polar coordinate. It was shown that the known helicity operator cannot be diagonalized in the system under consideration. To resolve the system of equations, we have applied the method by Fedorov – Gronskiy, which is based on the use of projective operators related to the third spin matrix S_3. We have constructed two types of the wave functions, the corresponding energy spectra are found in analytical form, these spectra are studied numerically as well.

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