УДК 512.542

# ОБ ОДНОМ ОБОБЩЕНИИ $\sigma$-ЛОКАЛЬНЫХ И БЭРА-ЛОКАЛЬНЫХ ФОРМАЦИЙ

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# ON ONE GENERALIZATION OF $\sigma$-LOCAL AND BAER-LOCAL FORMATIONS

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Все рассматриваемые в работе группы конечны, и $G$ – конечная группа. Пусть $\sigma = \{\sigma_i \mid i \in I\}$ – некоторое разбиение множества всех простых чисел $\mathbb{P}$. Тогда $\sigma(G) = \{\sigma_i \mid \sigma_i \cap \pi(G) \neq \varnothing\}$; $\sigma^+(G) = \{\sigma_i \mid G$ содержит главный фактор $H/K$, такой что $\sigma(H/K) = \{\sigma_i\}\}$. Группа $G$ называется: $\sigma$*-примарной*, если $G$ – $\sigma_i$-группа для некоторого $i$; $\sigma$*-разрешимой*, если каждый главный фактор из $G$ является $\sigma$-примарным. Символ $R_\sigma(G)$ обозначает произведение всех нормальных $\sigma$-разрешимых подгрупп из $G$. Главный фактор $H/K$ из $G$ называется: $\sigma$-центральным (в $G$), если произведение $(H/K) \rtimes (G/C_G(H/K))$ является $\sigma$-примарным; $\sigma_i$*-фактором*, если $H/K$ – $\sigma_i$-группа. Мы говорим, что $G$: $\sigma$*-нильпотентна*, если каждый главный фактор из $G$ $\sigma$-централен; *обобщенно* $\{\sigma_i\}$*-нильпотентна*, если каждый главный $\sigma_i$-фактор из $G$ $\sigma$-централен. Символ $F_{\{g\sigma_i\}}(G)$ обозначает произведение всех нормальных обобщенно $\{\sigma_i\}$-нильпотентных подгрупп из $G$. Мы называем произвольную функцию $f$ вида $f : \sigma \cup \{\varnothing\} \to \{$формации групп$\}$, где $f(\varnothing) \neq \varnothing$, *обобщенно формационной* $\sigma$*-функцией* и полагаем

$$BLF_\sigma(f) = (G \mid G/R_\sigma(G) \in f(\varnothing) \text{ и } G/F_{\{g\sigma_i\}}(G) \in f(\sigma_i) \text{ для всех } \sigma_i \in \sigma^+(G)).$$

Если для некоторой обобщенно формационной $\sigma$-функции $f$ имеет место $\mathfrak{F} = BLF_\sigma(f)$, то мы говорим, что класс $\mathfrak{F}$ является *Бэра-*$\sigma$*-локальным* и $f$ – *обобщенно* $\sigma$*-локальное определение* $\mathfrak{F}$. В данной работе описываются основные свойства, примеры и некоторые приложения Бэра-$\sigma$-локальных формаций.

***Ключевые слова***: *конечная группа*, *обобщенно формационная* $\sigma$*-функция*, *Бэра-*$\sigma$*-локальная формация*, *обобщенно* $\{\sigma_i\}$*-нильпотентная группа*, *произведение Гашюца*.

Throughout this paper, all groups are finite and $G$ is a group. Let $\sigma = \{\sigma_i \mid i \in I\}$ be some partition of the set of all primes $\mathbb{P}$. Then $\sigma(G) = \{\sigma_i \mid \sigma_i \cap \pi(G) \neq \varnothing\}$; $\sigma^+(G) = \{\sigma_i \mid G$ has a chief factor $H/K$ such that $\sigma(H/K) = \{\sigma_i\}\}$. The group $G$ is said to be: $\sigma$*-primary* if $G$ is $\sigma_i$-group for some $i$; $\sigma$*-soluble* if every chief factor of $G$ is $\sigma$-primary. The symbol $R_\sigma(G)$ denotes the product of all normal $\sigma$-soluble subgroups of $G$. The chief factor $H/K$ of $G$ is said to be: $\sigma$*-central* (in $G$) if $(H/K) \rtimes (G/C_G(H/K))$ is $\sigma$-primary; a $\sigma_i$*-factor* if $H/K$ is a $\sigma_i$-group. We say that $G$ is: $\sigma$*-nilpotent* if every chief factor of $G$ is $\sigma$-central; *generalized* $\{\sigma_i\}$*-nilpotent* if every chief $\sigma_i$-factor of $G$ is $\sigma$-central. The symbol $F_{\{g\sigma_i\}}(G)$ denotes the product of all normal generalized $\{\sigma_i\}$-nilpotent subgroups of $G$. We call any function $f$ of the form $f : \sigma \cup \{\varnothing\} \to \{$formations of groups$\}$, where $f(\varnothing) \neq \varnothing$, *a generalized formation* $\sigma$*-function* and we put

$$BLF_\sigma(f) = (G \mid G/R_\sigma(G) \in f(\varnothing) \text{ and } G/F_{\{g\sigma_i\}}(G) \in f(\sigma_i) \text{ for all } \sigma_i \in \sigma^+(G)).$$

If for some generalized formation $\sigma$-function $f$ we have $\mathfrak{F} = BLF_\sigma(f)$, then we say that the class $\mathfrak{F}$ is *Baer-*$\sigma$*-local* and $f$ is a *generalized* $\sigma$*-local definition of* $\mathfrak{F}$. In this paper, we describe basic properties, examples, and some applications of Baer-$\sigma$-local formations.

***Keywords***: *finite group*, *generalized formation* $\sigma$*-function*, *Baer-*$\sigma$*-local formation*, *generalized* $\{\sigma_i\}$*-nilpotent group*, *Gaschütz product*.

## 1 Base concept

Throughout this paper, all groups are finite and $G$ always denotes a finite group. Moreover, $\mathbb{P}$ is the set of all primes, $\pi \subseteq \mathbb{P}$ and if $n$ is an integer, then the symbol $\pi(n)$ denotes the set of all primes dividing $n$; as usual,

$$\pi(G)=\pi(|G|),$$

the set of all primes dividing the order of the group $G$; $\pi(\mathfrak{F})=\bigcup_{G\in\mathfrak{F}}\pi(G)$.

Following [1], $\sigma$ is some partition of $\mathbb{P}$, that is, $\sigma=\{\sigma_i \mid i\in I\}$, where $\mathbb{P}=\bigcup_{i\in I}\sigma_i$ and $\sigma_i\cap\sigma_j=\varnothing$ for all $i\neq j$. By analogy with the notations $\pi(n)$ and $\pi(G)$, we write

$$\sigma(n)=\{\sigma_i \mid \sigma_i\cap\pi(n)\neq\varnothing\}$$

and $\sigma(G)=\sigma(|G|)$ [2].

The group $G$ is said to be [3]: $\sigma$*-primary* if $G$ is a $\sigma_i$-group for some $i$; $\sigma$*-soluble* if $G=1$ or $G\neq 1$ and every chief factor of $G$ is $\sigma$-primary.

The chief factor $H/K$ of $G$ is said to be: $\sigma$*-central* (in $G$) if

$$(H/K)\rtimes(G/C_G(H/K))$$

is $\sigma$-primary; a $\sigma_i$*-factor* if $H/K$ is a $\sigma_i$-group. We say that $G$ is: $\sigma$*-nilpotent* if every chief factor of $G$ is $\sigma$-central [3]; $\{\sigma_i\}$*-nilpotent* if every chief factor $H/K$ of $G$ with $\sigma(H/K)\cap\sigma_i\neq\varnothing$ is $\sigma$-central; *generalized* $\{\sigma_i\}$*-nilpotent* if every chief $\sigma_i$-factor of $G$ is $\sigma$-central. We use $F_{\{\sigma_i\}}(G)$ (respectively $F_{\{g\sigma_i\}}(G)$) to denote the product of all normal $\{\sigma_i\}$-nilpotent (respectively generalized $\{\sigma_i\}$-nilpotent) subgroups of $G$.

In what follows, $\mathfrak{F}$ is a class of groups containing all identity groups; $G^{\mathfrak{F}}$ denotes the intersection of all normal subgroups $N$ of $G$ with $G/N\in\mathfrak{F}$; $G_{\mathfrak{F}}$ is the product of all normal subgroups $N$ of $G$ with $N\in\mathfrak{F}$. The class $\mathfrak{F}$ is said to be: a *formation* if for every group $G$ every homomorphic image of $G/G^{\mathfrak{F}}$ belongs to $\mathfrak{F}$; a *Fitting class* if for every group $G$ every normal subgroup of $G_{\mathfrak{F}}$ belongs to $\mathfrak{F}$.

The formation $\mathfrak{F}$ is called: *saturated* or *local* if $G\in\mathfrak{F}$ whenever $G/\Phi(G)\in\mathfrak{F}$; *solubly saturated* or *Baer-local* if $G\in\mathfrak{F}$ whenever

$$G/\Phi(R(G))\in\mathfrak{F},$$

where $R(G)$ is the *radical* of $G$, that is, the largest normal soluble subgroup of $G$; (*normally*) *hereditary* if $H\in\mathfrak{F}$ whenever $H\leq G\in\mathfrak{F}$ (respectively whenever $H\trianglelefteq G\in\mathfrak{F}$). The study of the $\sigma$-properties of the group, that are, its properties depending on the choice of the partition $\sigma$ of $\mathbb{P}$ leads in some situations to the need to find and study appropriate $\sigma$-modifications of these concepts.

Recall that any function $f$ of the form

$$f:\sigma\rightarrow\{\text{formations of groups}\}$$

is called a *formation* $\sigma$*-function* [4] and we put

$$LF_\sigma(f)=(G\mid G=1 \text{ or } G\neq 1 \text{ and } G/F_{\{\sigma_i\}}(G)\in f(\sigma_i) \text{ for all } \sigma_i\in\sigma(G)).$$

If for some formation $\sigma$-function $f$ we have $\mathfrak{F}=LF_\sigma(f)$, then we say, following [4], that the class $\mathfrak{F}$ is $\sigma$*-local* and $f$ is a $\sigma$*-local definition* of $\mathfrak{F}$.

The basic properties and various applications of $\sigma$-local formations were discussed in the papers [4]–[8].

In this paper, we introduce and study various classes of *Baer-*$\sigma$*-local formations*, which simultaneously generalize $\sigma$-local and Baer-local formations (see [9, Chapter IV] or [10, Chapter 3]). In particular, we get that the set of all Baer-$\sigma$-local formations containing all nilpotent groups forms a subsemigroup of the semigroup of all formations $G\mathfrak{G}$ [11, p. 67] and the set of all Baer-$\sigma$-local formations containing all $\sigma$-nilpotent groups is a right ideal in $G\mathfrak{G}$.

Let

$$\sigma^+(G)=\{\sigma_i \mid G \text{ has a chief factor } H/K \text{ such that } \sigma(H/K)=\{\sigma_i\}\},$$

$\sigma(\mathfrak{F})=\bigcup_{G\in\mathfrak{F}}\sigma(G)$ and $\sigma^+(\mathfrak{F})=\bigcup_{G\in\mathfrak{F}}\sigma^+(G)$.

We call any function $f$ of the form

$$f:\sigma\cup\{\varnothing\}\rightarrow\{\text{formations of groups}\},$$

where $f(\varnothing)\neq\varnothing$, a *generalized formation* $\sigma$*-function* [12] and we put, following [12],

$$BLF_\sigma(f)=(G\mid G/R_\sigma(G)\in f(\varnothing) \text{ and } G/F_{\{g\sigma_i\}}(G)\in f(\sigma_i) \text{ for all } \sigma_i\in\sigma^+(G)).$$

The symbol Supp($f$) denotes the *support* of $f$, that is, the set of all $\sigma_i$ such that $f(\sigma_i)\neq\varnothing$.

***Definition***. If for some generalized formation $\sigma$-function $f$ we have $\mathfrak{F}=BLF_\sigma(f)$, then we say that the class $\mathfrak{F}$ is *Baer-*$\sigma$*-local* and $f$ is a *generalized* $\sigma$*-local definition* of $\mathfrak{F}$.

**2 Main results**

We use $\mathfrak{G}_{\Pi^+}$ to denote the class of all groups $G$ with

$$\sigma^+(G)\subseteq\Pi\subseteq\sigma.$$

Note that $\mathfrak{G}_{\varnothing^+}$ is the class of all groups $G$ with

$$\sigma^+(G)=\varnothing;$$

$\mathfrak{G}_{\{g\sigma_i\}}$ denotes the class of all generalized $\{\sigma_i\}$-nilpotent groups, $\mathfrak{S}_\sigma$ is the class of all $\sigma$-soluble groups.

If $\mathfrak{M}$ and $\mathfrak{H}$ are non-empty formations, then $\mathfrak{M}\mathfrak{H}$ is the class of groups $G$ such that for some normal subgroup $N$ of $G$ we have $G/N\in\mathfrak{H}$ and $N\in\mathfrak{M}$.

Our first two results are basic.

***Proposition* 2.1** [12, Proposition 1.2]. *Let* $\mathfrak{F} = BLF_\sigma(f)$ *and* $\Pi = \mathrm{Supp}(f)$.

(1) $\mathfrak{F}$ *is a non-empty formation and* $\Pi = \sigma^+(\mathfrak{F})$.

(2) $G \in \mathfrak{F}$ *if and only if*

$$G \in \mathfrak{S}_\sigma f(\varnothing) \text{ and } G \in \mathfrak{G}_{\{g\sigma_i\}} f(\sigma_i)$$

*for all* $\sigma_i \in \sigma^+(G)$.

(3)

$$\mathfrak{F} = \begin{cases} \left(\bigcap_{\sigma_i \in \Pi} \mathfrak{G}_{\{g\sigma_i\}} f(\sigma_i)\right) \cap \mathfrak{S}_\sigma f(\varnothing) \cap \mathfrak{G}_{\Pi^+} & \text{if } \Pi \neq \varnothing, \\ \mathfrak{S}_\sigma f(\varnothing) \cap \mathfrak{G}_{\Pi^+} & \text{if } \Pi = \varnothing. \end{cases}$$

Note that the class $\mathfrak{G}_{\{g\sigma_i\}}$ is a Fitting formation, so the product $\mathfrak{G}_{\{g\sigma_i\}} f(\sigma_i))$ is also a Fitting formation for all

$$\sigma_i \in \Pi = \sigma^+(\mathfrak{F}).$$

Hence from Proposition 2.1 we get the following useful fact.

***Corollary* 2.2.** *Let* $\mathfrak{F} = BLF_\sigma(f)$ *be a Baer-σ-local formation. If* $f(a)$ *is normally hereditary* (*respectively a Fitting formation*) *for all* $a \in \sigma \cup \{\varnothing\}$, *then* $\mathfrak{F}$ *is also normally hereditary* (*respectively a Fitting formation*).

Now we show that every Baer-σ-local formation possesses a generalized σ-local definition for which the inverse of this corollary holds.

A (generalized) formation σ-function *f* is said to be: *integrated* if

$$f(\sigma_i) \subseteq LF_\sigma(f)$$

for all $i$ (if, respectively, $f(a) \subseteq BLF_\sigma(f)$ for all $a \in \sigma \cup \{\varnothing\}$); *full* if

$$f(\sigma_i) = \mathfrak{G}_{\sigma_i} f(\sigma_i)$$

for all $i$ (if, respectively, $f(\varnothing) = BLF_\sigma(f)$ and $f(\sigma_i) = \mathfrak{G}_{\sigma_i} f(\sigma_i)$ for all *i*).

***Theorem* 2.3** [12, Theorem 1.4]. *Let* $\mathfrak{F}$ *be a Baer-σ-local formation. Then* $\mathfrak{F}$ *has a unique full integrated generalized σ-local definition F. Moreover,*

(*i*) $F(\sigma_i) = \mathfrak{G}_{\sigma_i}(f(\sigma_i) \cap \mathfrak{F})$ *for every generalized σ-local definition f of* $\mathfrak{F}$ *and for all* $\sigma_i \in \sigma^+(\mathfrak{F})$, *and*

(*ii*) *If* $\mathfrak{F}$ *is normally hereditary* (*respectively a Fitting formation*), *then* $F(a)$ *is normally hereditary* (*respectively a Fitting formation*) *for all* $a \in \sigma \cup \{\varnothing\}$.

If $F$ is a full integrated σ-function and $\mathfrak{F} = BLF_\sigma(F)$ (respectively $\mathfrak{F} = LF_\sigma(F)$), then we say that $F$ is the *canonical generalized σ-local definition* (respectively *canonical σ-local definition*) of $\mathfrak{F}$.

If $f$ and $g$ be generalized formation σ-functions such that $f(a) \subseteq g(a)$ for all $a \in \sigma \cup \{\varnothing\}$, then we write $f \leq g$. It is clear that if $f \leq g$, then

$$BLF_\sigma(f) \subseteq BLF_\sigma(g).$$

The second important property of the canonical generalized σ-local definition of a Baer-σ-local formation is associated with the following fact, which is also a corollary of Proposition 2.1.

***Corollary* 2.4.** *Let* $\mathfrak{F}_i = BLF_\sigma(F_i)$, *where* $F_i$ *is the canonical generalized σ-local definition of* $\mathfrak{F}_i$, $i = 1, 2$. *Then* $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ *if and only if* $F_1 \leq F_2$.

***Theorem* 2.5** [12, Theorem 1.6]. *Let* $\mathfrak{F} = LF_\sigma(f)$ *be a σ-local formation. Then* $\mathfrak{F} = BLF_\sigma(g)$, *where* $g(\varnothing) = \mathfrak{F}$ *and* $g(\sigma_i) = f(\sigma_i)$ *for all i. Moreover, if h is any integer generalized σ-local definition of* $\mathfrak{F}$, *then* $\mathfrak{F} = LF_\sigma(t)$, *where t is the restriction of h on* σ.

In view of Theorem 2.5, we get from Theorem 2.3 the following known results.

***Corollary* 2.6** [7, Proposition 1.6]. *Every σ-local formation has the unique canonical σ-local definition.*

In the case when $\sigma = \{\{2\}, \{3\}, \ldots\}$ we get from Theorem 2.3 and Corollary 2.6 the following results.

***Corollary* 2.7** [1, Chapter I, Theorem 3.2]. *Every Baer-local formation has the unique canonical generalized local definition.*

***Corollary* 2.8** [1, Chapter I, Theorem 3.3]. *Every local formation has the unique canonical local definition.*

***Corollary* 2.9** [1, Chapter I, Theorem 4.7 and 4.10]. *Let* $\mathfrak{F}$ *be the generalized local definition of Baer-local formation* $\mathfrak{F}$. *If* $\mathfrak{F}$ *is a Fitting formation, then* $F(p)$ *is a Fitting formation for all primes p.*

Before continuing, consider a few examples.

***Example* 2.10.** (i) In the classical case, when $\sigma = \sigma^1 = \{\{2\}, \{3\}, \ldots\}$ (we use here the notation in [2]): $\mathfrak{F}$ is a local formation if and only if it is a $\sigma^1$-local formation by [13, Chapter VI, Hilfssatz 7.4] (see also [9, Chapter IV, Theorem 3.2]) and $\mathfrak{F}$ is a Baer-local formation if and only if it is a Baer-$\sigma^1$-local formation by the results in [14].

(ii) Every Baer-σ-local formation $\mathfrak{F}$ which contains only σ-soluble groups is σ-local. Indeed, if $\mathfrak{F} = BLF_\sigma(f)$ and $h$ is a formation σ-function such that $h(\sigma_i) = f(\sigma_i)$ for all *i*, then $G \in \mathfrak{F}$ if and only if $G \in LF_\sigma(h)$ since for every σ-soluble group $G$ we have $F_{\{\sigma_i\}}(G) = F_{\{g\sigma_i\}}(G)$ for every $\sigma_i \in \sigma(G)$.

(iii) The group $G$ is called *σ-semisimple* [15] if either $G = 1$ or $G = A_1 \times \cdots \times A_t$ is the direct product of simple non-σ-primary groups $A_1, \ldots, A_t$. We use $\mathfrak{M}_\sigma$ to denote the class of all σ-semisimple groups. Let $f(\varnothing)=\mathfrak{M}_\sigma$ and $f(\sigma_i)=\varnothing$ for all $i$. Then

$$BLF_\sigma(f)=\mathfrak{S}_\sigma\mathfrak{M}_\sigma\cap\mathfrak{G}_{\varnothing^+}=\mathfrak{M}_\sigma$$

by Proposition 2.1 (3). Hence $\mathfrak{M}_\sigma$ is a Baer-$\sigma$-local formation.

(iv) The group $G$ is: $\sigma$-*quasinilpotent* [16] if it has a normal subgroup $Z$ such that $G/Z$ is $\sigma$-semisimple and every chief factor of $G$ below $Z$ is $\sigma$-central in $G$; *generalized* $\sigma$-*nilpotent* if every $\sigma$-primary chief factor of $G$ is $\sigma$-central in $G$. We use $\mathfrak{N}_\sigma^*$ and $\mathfrak{N}_{g\sigma}$ to denote the class of all $\sigma$-quasinilpotent groups and the class of all generalized $\sigma$-nilpotent groups, respectively.

L.A. Shemetkov proved [17] that the class of all quasinilpotent groups $\mathfrak{N}^*=\mathfrak{N}_{\sigma^1}^*$ is a Baer-local formation. Now we show that the class $\mathfrak{N}_\sigma^*$ is a Baer-$\sigma$-local formation for each partition $\sigma$ of $\mathbb{P}$. Indeed, let $f$ be a generalized formation $\sigma$-function such that $f(\varnothing)=\mathfrak{M}_\sigma$ is the class of all $\sigma$-semisimple groups and $f(\sigma_i)=\mathfrak{N}_{\sigma_i}$ for all $i$. It is clear that $\sigma^+(\mathfrak{N}_\sigma^*)=\sigma$ and so, by Proposition 2.1 (3), we have

$$\left(\bigcap_{\sigma_i\in\sigma}\mathfrak{G}_{\{g\sigma_i\}}\mathfrak{G}_{\sigma_i}\right)\cap\mathfrak{S}_\sigma\mathfrak{M}_\sigma\cap\mathfrak{G}_{\sigma^+}=$$

$$=\left(\bigcap_{\sigma_i\in\sigma}\mathfrak{G}_{\{g\sigma_i\}}\right)\cap\mathfrak{S}_\sigma\mathfrak{M}_\sigma=\mathfrak{N}_{g\sigma}\cap\mathfrak{S}_\sigma\mathfrak{M}_\sigma=\mathfrak{N}_\sigma^*$$

since $\mathfrak{G}_{\{g\sigma_i\}}\mathfrak{G}_{\sigma_i}=\mathfrak{G}_{\{g\sigma_i\}}$. Therefore the class $\mathfrak{N}_\sigma^*$ is a Baer-$\sigma$-local formation.

The *Gaschütz product* $\mathfrak{M}\circ\mathfrak{H}$ of the formations $\mathfrak{M}$ and $\mathfrak{H}$ is defined as folows: $G\in\mathfrak{M}\circ\mathfrak{H}$ if and only if $G^{\mathfrak{H}}\in\mathfrak{M}$ (by defitnition $\mathfrak{M}\circ\mathfrak{H}=\varnothing$ in the case when $\mathfrak{H}=\varnothing$). It is easy to verify that $\mathfrak{M}\circ\mathfrak{H}$ is a formation, and if $\mathfrak{M}$ is normally hereditary, then $\mathfrak{M}\mathfrak{H}=\mathfrak{M}\circ\mathfrak{H}$. Moreover, for every three formations $\mathfrak{M}$, $\mathfrak{H}$ and $\mathfrak{F}$ we have

$$(\mathfrak{M}\circ\mathfrak{H})\circ\mathfrak{F}=\mathfrak{M}\circ(\mathfrak{H}\circ\mathfrak{F})$$

(see [11, Chapter II] or [9, Chapter IV]). Therefore the set $G\mathfrak{G}$ [11, p. 67], of all formations, forms a semigroup with respect to the operation $\circ$.

Now we give the conditions under which the Gaschütz product of two formations is Baer-$\sigma$-local.

***Theorem* 2.11** [12, Theorem 1.12]. *Let* $\mathfrak{M}=BLF_\sigma(m)$ *and* $\mathfrak{H}=BLF_\sigma(h)$, *where* $m$ *and* $h$ *are integrated. Suppose that* $\mathfrak{M}$ *contains each* $p$-*group for all* $p\in\pi(\mathfrak{M})$. *Then* $\mathfrak{M}\circ\mathfrak{H}=$ $=BLF_\sigma(f)$, *where*

$$f(a)=\begin{cases} m(\sigma_i)\circ\mathfrak{H} & \text{if } a=\sigma_i\in\sigma^+(\mathfrak{M}),\\ h(\sigma_i) & \text{if } a=\sigma_i\in\sigma\setminus\sigma^+(\mathfrak{M}),\\ m(\varnothing)\circ\mathfrak{H} & \text{if } a=\varnothing.\end{cases}$$

As a first application of this result, we get the following

***Theorem* 2.12** [12, Theorem 1.13]. *The set of all Baer-*$\sigma$*-local formations* $\mathfrak{F}$ *containing all nilpotent* $\pi$-*groups, where* $\pi=\pi(\mathfrak{F})$, *forms a subsemigroup of the semigroup of all formations* $G\mathfrak{G}$.

From Theorem 2.11 we get also the following known result.

***Corollary* 2.13** [11, Chapter 2, Theorem 7.9]. *The Gaschütz product* $\mathfrak{M}\circ\mathfrak{H}$ *of any two Baer-local formations* $\mathfrak{M}$ *and* $\mathfrak{H}$, *where* $\mathfrak{M}$ *contains each p-group for all* $p\in\pi(\mathfrak{M})$, *is also a Baer-local formation.*

***Theorem* 2.14** [12, Theorem 1.15]. *Let* $\mathfrak{M}=BLF_\sigma(m)$ *and let* $\mathfrak{H}$ *be a non-empty formation with* $\sigma^+(\mathfrak{H})\subseteq\sigma^+(\mathfrak{M})$, *where m is integrated. Then*

$$\mathfrak{M}\circ\mathfrak{H}=BLF_\sigma(f),$$

*where*

$$f(a)=\begin{cases} m(\sigma_i)\circ\mathfrak{H} & \text{if } a=\sigma_i\in\sigma^+(\mathfrak{M}),\\ \varnothing & \text{if } a=\sigma_i\in\sigma\setminus\sigma^+(\mathfrak{M}),\\ m(\varnothing)\circ\mathfrak{H} & \text{if } a=\varnothing.\end{cases}$$

From Theorem 2.14 we get the following

***Theorem* 2.15** [12, Theorem 1.16]. *The set of all Baer-*$\sigma$*-local formations containing all* $\sigma$-*nilpotent groups forms a right ideal in the semigroup of all formations* $G\mathfrak{G}$.

From Theorem 2.14 we get also the following known result.

***Corollary* 2.16** [11, Chapter 2, Theorem 7.10]. *The Gaschütz product* $\mathfrak{M}\circ\mathfrak{H}$ *of any two formations* $\mathfrak{M}$ *and* $\mathfrak{H}$, *where* $\mathfrak{M}$ *is a Baer-local formation containing all nilpotent groups, is also a Baer-local formation.*

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*Поступила в редакцию* 01.11.19.