Characterization of simulation models of complex systems

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Abstract. *The article addresses an investigation of characteristics of simulation models suggesting the following sequence of actions: assessment of simulation error due to imperfections of pseudorandom-number generators used in simulation models; determination of simulation time needed to achieve the desired simulation precision; assessment of resiliency of simulation results (Y) at different changes of parameters (X) and at given values of variables (G); assessment of sensitivity of simulation results to changes of model parameters (X) and assessment of simulation raw data (G).*

I. Introduction

Characterization of simulation models of complex systems shall determine the following aspects: simulation precision (ε_n), simulation transition period (T_{tp}), simulation resiliency (UST). Any testing of a simulation model of complex systems starts from these procedures. Let's review the technology of implementation of these three procedures.

Generators of pseudorandom numbers, uniformly distributed within the interval [0, 1], known as base generators, are used in any probabilistic simulation model of complex systems. The simulation system provides the user with several different base generators. These generators and the very probabilistic nature of the simulation model are the sources of imitation errors.

To check the quality of the generators, there are descriptions of tests of such base generators in the documents on the simulation system. Sometimes, it is a challenge to choose, which of the base generators suits better for the particular use.

II. Procedure for simulation error determination

The following procedure for determination of simulation error approved on different simulation models is suggested. A series of runs of simulation models ($k \leq 30$) with same values of parameters and variables (X and G), but with different initial values of base generators (ξ_{okm}, k – run number, m – base generator number), forms in the midpoint of the change region of control parameters $\{X\}$. [1]

The values of the n^{th} component of response vector Y_{nk} are determined in each run. As the result, a fetch of responses is formed $\{Y_{nk}\}$. The fetch is assessed for the expected value and the dispersion ($\overline{Y}_n$, D_n).

The confidence interval of determination of the actual value of the expected value of the n^{th} component (Y_{hn}) is established. Let's assume normality of deviation distribution from $\overline{Y}_{hk}$ to $\overline{Y}^*_{hn}$.

Since the sizes of fetches are small ($N \leq 30$), t-statistics with Student's distribution is used to determine confidence intervals (1).

$$t = \left(\overline{Y}_h - \overline{Y}^*_{hn}\right)\sqrt{\frac{N-1}{D_n}}, \tag{1}$$

With the significance level ($\alpha = 0.05$) and probability 0.95 taken as an example Y^*_{hn} is assumed within:

$$\overline{Y}_n - t_{0.05}\sqrt{\frac{D_h}{N-1}} \leq Y_{hn} < \overline{Y}_n + t_{0.05}\sqrt{\frac{D_h}{N-2}}, \tag{2}$$

where $t_{0,05}$ is the t-statistics value at (N–1) degrees of freedom and significance level α =0.05.

Thus, the error may be determined for each component by the following formulae: (3)

$$\alpha_n = t_{0.05}\sqrt{\frac{D_h}{N-2}} \tag{3}$$

It shall be noted, that the minimum number of simulation experiments shall be equal to 30. In this case, the error with confidence $\beta = 0.95$ is $\varepsilon = 0.753\ S_n$ (were S_n is the mean square value of the fetch with the size of $N = 30$), which is the upper value of the error due to probabilistic nature of the simulation model and use of base generators of pseudorandom values.

Should the error be too high, the investigator shall review the composition of generators and the procedure of their use in the simulation model of complex systems. These cases require use of a higher number of base generators with a guaranteed quality of simulation of ξ_{okm}.

III. Approbation of assessment of response sensitivity of imitation models of complex systems

Assessment of sensitivity of the results of simulation modeling consists of determination of the range of changes in the responses of the simulation model $\{Y_j\}$ depending on the changes in the set of parameters of the simulation model $\{X_k\}$ over the entire range of their change.

Depending on the range of changes of the responses {Yj}, the future strategy for operating the simulation model is determined. If at insignificant amplitude of vector components (even in the entire change range from X_{umin} to X_{max}) the response Y_j changes insignificantly, it means that the representation precision Y_j in the simulation model is of small importance. Then this response shall not be used as the key one for simulation experiment designing, or may even be discarded at implementation of the series of simulation experiments on the simulation model. At the same time, if the response Y_j is highly sensitive to changes of the parameter X_k, it directly confirms the need to represent it in the simulation model with the maximum possible precision.

The procedure for determination of sensitivity of the simulation results is as follows. Let each q^{th} component of the parameter vector $\{X_q\}$ deviate from its value at the central point in both directions by the length of the selected interval of its change (minX_q, maxX_q). And let the remaining components of the vector $\{X_q\}$ remain unchanged and correspond to the central point in the parameter space.

A pair of model experiments is performed for the specified values of the parameter vector $\{X_q\}$ and the values of the j^{th} response, denoting them as Y_j^- (for X_{qmin}) and Y_j^- (for X_{qmax}), are calculated. The coefficient of sensitivity of response Y_j to the changes of parameter X_q is calculated by the formula:

$$\delta Y_{kj} = \frac{\left|Y_j^+ - Y_j^-\right| \cdot 2 \cdot 100\%}{\left(Y_j^+ + Y_j^-\right)}, \tag{4}$$

where Y_j^+ and Y_j^- are the changes of response Y_j at change of parameter X_q respectively to the maximum and the minimum of its changes within the entire range. Then the sensitivity matrix of responses $\left\| \delta Y_{kj} \right\|$ is constructed, where the q^{th} line corresponds to the changes of the component of the parameter vector X_q in the dynamics of changes of its values, while the j^{th} column corresponds to the number of the component of the vector of the response of the simulation model. δY_{kj} is calculated by the following formulae (4).

Then the content of the matrix $\left\| \delta Y_{kp} \right\|$ is analyzed as follows. Each component of the matrix is verified for trueness of inequality:

$$\delta Ykj \leq \xi u\%, \tag{5}$$

where ξ_u % is the maximum error of simulation of the variants of the simulation model of the complex system. The Boolean matrix $\left\| \gamma_{kj} \right\|$ is formed, where:

$$\gamma_{kj} = \begin{cases} 0, when\ the\ inequelity\ is\ true \\ \hline 1, otherwise \end{cases}, \tag{6}$$

If the k^{th} line of the matrix is zero, it means that the component of the parameter vector in the given variant of the complex system simulation model affects almost all components of the response vector $\{Y_j\}$ and the parameter in this study may be discarded. As the result, the number of the vector component of the parameter of the complex system simulation model is minimized. [2]

While, if the j^{th} column of the matrix γ is equal to zero, it means that the given study may discard the j^{th} component of the response vector of the simulation model. Thus, the number of responses of the complex system simulation model at the given composition of the remaining variables of the simulation $\{G\}$ is minimized.

It's useful to range all responses to have the responses with the maximum values δY_{kj} on the left side. Let's name all responses, for which the inequality (5) is not satisfied, the main ones, and the remaining responses - the minor ones (for the specific value of the parameter vector component).

The parameters for the Y_j^{th} component of the response vector are ranged in a similar manner. Similarly, let's name all parameters, for which the inequality (5) is satisfied, significant, and the remaining parameters - insignificant.

Obviously, only the main parameters shall be involved in experiment designing at subsequent stages of operation of the complex system simulation model (the investigator shall pay attention to cross-coupling effect of the main responses with the significant parameters).

IV. Conclusion

This article considers the following aspects of the technology of process characterization of the complex system simulation model: assessment of the precision of the simulation model program; determination of the confidence interval of the mathematical expectation of the nth response component; determination of the length of the simulation transition period, assessment of simulation stability; implementation of the rules for automatic termination of the simulation.

Steps are proposed for the technology of sensitivity assessment of the complex system simulation model covering calculation of coefficients of response sensitivity to parameter variations; development of the matrix of coefficients of sensitivity; rejection of the composition of responses and parameters using the coefficients of sensitivity of the simulation model.

References

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